High-order well-balanced numerical methods for hyperbolic balance laws

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The goal of this presentation is to review the design of high-order well-balanced numerical methods for hyperbolic balance laws. More precisely, we focus on the design of high-order finite volume numerical methods, based on a reconstruction operator that preserves all the stationary solutions or a relevant subset of them, of 1d hyperbolic balance laws:

$$U_t + F(U)_x = S(U)\sigma_x, \tag{1}$$

where $U(x,t)$ takes values in $\Omega \subset \mathbb{R}^N$; $F : \Omega \to \mathbb{R}^n$ is the flux function; $S : \Omega \to \mathbb{R}^N$; and $\sigma \colon \mathbb{R} \to \mathbb{R}$ is a known function (possibly the identity $\sigma(x) = x$).

First, we recall the form of a standard high-order finite volume scheme based on a reconstruction operator and then we present a general procedure to extend a standard method so that it is well-balanced in the case where σ is a continuous function. We also discuss the main difficulties that appear when σ is a non-smooth function and the extension to 2D domains. Finally, several applications to well-known systems like the 1d Euler with gravity or the shallow-water system over the sphere will be presented.

References

[1] M. J. Castro, T. Morales de Luna, and C. Parés, Well-balanced schemes and path-conservative numerical methods. In *Handbook of Numerical Analysis* 18, 1315-1341, Elsevier 2017.